NOTIZEN

Non-local Generalization of the Lorentz-Dirac Equation and the Problem of Runaway Solutions

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A new, covariant equation of motion for the radiating electron of finite extension is proposed. This new equation excludes the notorious runaway solutions and pre-acceleration.

Recently a finite-size model of the classical radiating electron has been developed[1, 2], which reduces to the famous Lorentz-Dirac theory[3, 4], if a certain point limit * is performed. In this finite-size model no runaway solutions can occur, as is demonstrated in a future paper; and since the finite size of the electron has led to a non-local equation of motion for the radiating electron, one might rise the question whether the absence of runaway solutions is a common feature of all possible nonlocal generalizations of the Lorentz-Dirac equation

$$m\,c^2\,\dot u^\lambda = K^\lambda + \tfrac{2}{3}\,Z^2\,[\ddot u^\lambda + (\dot u\,\dot u)\,u^\lambda] \qquad (1)$$

(four-velocity $u^\lambda = \mathrm{d}z^\lambda/\mathrm{d}s$; $u^\lambda u_\lambda = 1$). This equation is assumed to describe the point-like radiating electron despite the emergence of such unphysical effects as pre-acceleration and runaway solutions[4]. If the question mentioned above could be answered in the positive, one had to conclude that there must be considerable doubt on the Lorentz-Dirac Equation (1), because a consistent point limit such as (1) should not exhibit features (i. e. runaway solutions), which are completely absent in the more general theory of finite extension!

We do not want to clarify here our question most generally, but we shall look for a further, natural and simple non-local generalization of Eq. (1) and shall then proof that also this generalization does not exhibit runaway solutions:

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* This point limit does not seem to be a consistent one, though generally accepted in literature [cf. equations (2) to (5)].

Consider the equation

$$m\,c^2[\hat{\dot u}^\lambda - (\hat{\dot u}\,u)\,u^\lambda] = K^\lambda \qquad (2)$$

with

$$u^\lambda = u^\lambda(s)\,; \quad \dot u^\lambda = \dot u^\lambda(s) \equiv \mathrm{d}u^\lambda/\mathrm{d}s\,; \qquad (3)$$
$$\hat{\dot u}^\lambda \equiv \dot u^\lambda(s - \varDelta s)$$

and $\varDelta s$ being a fixed proper-time interval, as in reference[1]. First, we have to show the emergence of Eq. (1), if we expand in Eq. (2) with respect to $\varDelta s$. With

$$\hat{\dot u}^\lambda \cong \dot u^\lambda - \varDelta s\,\ddot u^\lambda + \dots \qquad (4)$$

and $(u\,\ddot u) = -(\dot u\,\dot u)$ one finds from (2)

$$m\,c^2\,\dot u^\lambda = K^\lambda + m\,c^2\,\varDelta s\,[\ddot u^\lambda + (\dot u\,\dot u)\,u^\lambda]\,. \qquad (5)$$

Therefore we have the constraint for $\varDelta s$

$$m\,c^2\,\varDelta s = \tfrac{2}{3}\,Z^2\,. \qquad (6)$$

Defining the classical electron radius r_c by

$$m\,c^2 = Z^2/2\,r_\mathrm{c}\,, \qquad (7)$$

the numerical value of $\varDelta s$ is given by ($r_\mathrm{c} \approx 1.4 \cdot 10^{-13}$ cm)

$$\varDelta s = \tfrac{4}{3}\,r_\mathrm{c}\,. \qquad (8)$$

Next, we have to show the non-existence of runaway solutions in Equation (2). To this end, put $\{K^\lambda\} = 0$ and multiply Eq. (2) with $\{\hat{\dot u}_\lambda\}$ to obtain

$$(\hat{\dot u}\,\hat{\dot u}) = (u\,\hat{\dot u})^2\,. \qquad (9)$$

Since the four-acceleration $\{\dot u^\lambda\}$ is always a space-like vector $[(\hat{\dot u}\,\hat{\dot u}) \leqq 0]$ and the square of the scalar product $(\hat{\dot u}\,u)$ is always positive $[(\hat{\dot u}\,u)^2 \geqq 0]$ for an arbitrary world line, Eq. (9) requires

$$(\hat{\dot u}\,\hat{\dot u}) = (u\,\hat{\dot u})^2 = 0 \qquad (10)$$

in every point of the force-free world line. This requirement can only be satisfied in the case of uniform motion. So the extended particle, described by (2), can only move with constant four-velocity in force-free regions. This was set out to proof.

As a non-local generalization of the usual Lorentz-force $K^\lambda \to Z\,F^{\mu\lambda}\,u_\mu$ one might assume now

$$K^\lambda = Z\,F^{\mu\lambda}(s - \varDelta s)\,u_\mu \equiv Z\,\hat F^{\mu\lambda}\,u_\mu\,. \qquad (11)$$

Inserting (11) into (2) and multiplying with $\{\hat{\dot u}_\lambda\}$ yields

$$m\,c^2\,[(\hat{\dot u}\,\hat{\dot u}) - (\hat{\dot u}\,u)^2] = Z\,\hat F^{\mu\lambda}\,u_\mu\,\hat{\dot u}_\lambda\,. \qquad (12)$$

Assume now, that in a certain space-time point $\{\hat{z}^\lambda\}$ the external field strength $F^{\mu\lambda}$ vanishes

$$F^{\mu\lambda}(\hat{z}) \equiv \hat{F}^{\mu\lambda} = 0 . \qquad (13)$$

Then we conclude from (12) and (13) in a similar way as above

$$\dot{u}^\lambda(\hat{z}) \equiv \hat{\dot{u}}^\lambda = 0 . \qquad (14)$$

So one finds that, whenever the external force on the particle vanishes at a certain time, the four-acceleration of the particle vanishes at the same time, too. This excludes pre-acceleration in the usual sense. The new Eq. (2) can therefore be looked upon as a most natural generalization of the ordinary equation of motion for a non-radiating point particle

$$m\, c^2\, \dot{u}^\lambda = K^\lambda . \qquad (15)$$

Indeed, we have only to replace $\{\dot{u}^\lambda\}$ in Eq. (15) by the orthogonal component of the "advanced" four-acceleration $\{\hat{\dot{u}}^\lambda\}$ with respect to $\{u^\lambda(s)\}$.

The new, non-local equation of motion (2) seems to be very promising in various aspects and shall be investigated further.

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